Assignment 4

Modern Theory of Markov Chains

Due: 19.03.2013

1 (Non-reversibility)**.** Give an example of an irreducible finite-state Markov chain whose transition probabilities are all positive (i.e., $K(a,b) > 0$ for every two states a and b) and which is *not* reversible.

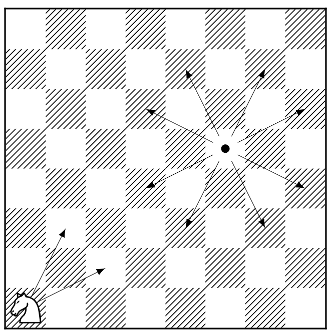
2 ("Almost every" Markov chain has a random walk hidden inside.)**.**

a) Show that every finite-state Markov chain whose transition probabilities are rational is a projection of a random walk on a directed graph (potentially with loops but no multiple edges). [See Section 2.3.1 of the book for what we mean by "projection".]

b) Show that every reversible Markov chain can be viewed as a random walk on a weighted (undirected) graph (potentially with loops but no multiple edges). [A weighted graph is a graph in which every edge is given a non-negative weight. A random walk on a weighted graph goes from a vertex to a neighbouring vertex with probability proportional to the weight of the connecting edge.]

(BONUS) c) Show that every finite-state reversible Markov chain whose transition probabilities are rational is a projection of a random walk on an (unweighted, undirected) graph (without loops or multiple edges).

3 (Knight on a chessboard)**.** A knight is doing a random walk on a chessboard, starting from a corner, and at each step following one of its legal moves at random. What is the expected number of moves it takes for the knight to return to its original position? (A legal move of the knight is L-shaped: two squares horizontally and one vertically, or vice versa.)



4 (Random walk with drift towards origin)**.** A particle is doing a random walk on $\mathbb{Z}$ with a slight bias towards the origin. When at a position $k \neq 0$, it either moves one step towards the origin (with probability $p > 1/2$), or moves one step away from the origin (with probability $1-p$). If at the origin, the particle moves randomly in one of the two directions.

(BONUS)

a) Perform a computer experiment to see how the distribution of the position of the particle evolves with time.

b) Does the model have a stationary distribution? Prove your claim.

c) What can you say about the distribution of the particle in the long run based on whether it starts from position 0 or position 100?