Markov Chains

Basics

In[1]:=
```
MarkovStep[K_, s_] := RandomChoice[K[[s]] -> Range[Dimensions[K][[2]]]]
MarkovChain[K_, s_, t_] := NestList[MarkovStep[K, #] &, s, t]  (* A sample of t
  steps from a Markov chain with transition matrix K starting from state s *)
```

In[3]:=
```
TransitionsOf[K_] := Flatten[Table[If[K[[i, j]] > 0, i ð j, {}],
   {i, Dimensions[K][[1]]}, {j, Dimensions[K][[2]]}]]
 (* The edges of the directed graph of probable transitions
  associated to the transition matrix K *)
```

Examples

In[4]:=
$$RWC_5 = \begin{pmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0.5 & 0 \end{pmatrix};$$ (* Simple random walk on C_5 *)

$$K_1 = \begin{pmatrix} 0.4 & 0.3 & 0 & 0 & 0.3 \\ 0.5 & 0 & 0.4 & 0.1 & 0.0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.1 & 0 & 0.5 & 0 & 0.4 \\ 0.4 & 0.1 & 0 & 0.5 & 0 \end{pmatrix};$$

Visualization

Active State

In[6]:=
```
ActiveStateStyle[x_] := {_ → Gray, x → Yellow}
MarkovActivePlot[K_, x_] :=
 Graph[TransitionsOf[K], VertexStyle → ActiveStateStyle[x], VertexSize → Medium]
```

Distribution

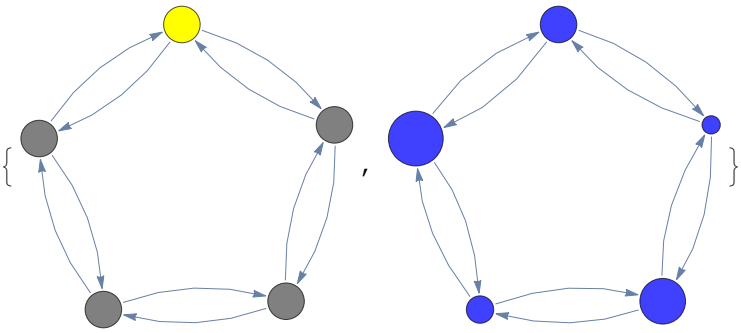
In[8]:=
```
DistributionStateStyle[u_] := Table[i → u[[i]], {i, 1, Length[u]}]
MarkovDistributionPlot[K_, u_] := Graph[TransitionsOf[K],
  VertexStyle → RGBColor[0.25, 0.25, 1], VertexSize → DistributionStateStyle[u]]
```

Example

In[10]:=
```
u = {0.1, 0.2, 0.3, 0.15, 0.25};
```

In[11]:=
```
{MarkovActivePlot[RWC5, 2], MarkovDistributionPlot[RWC5, u]}
```

Out[11]=



Evolution

Evolution of the distribution

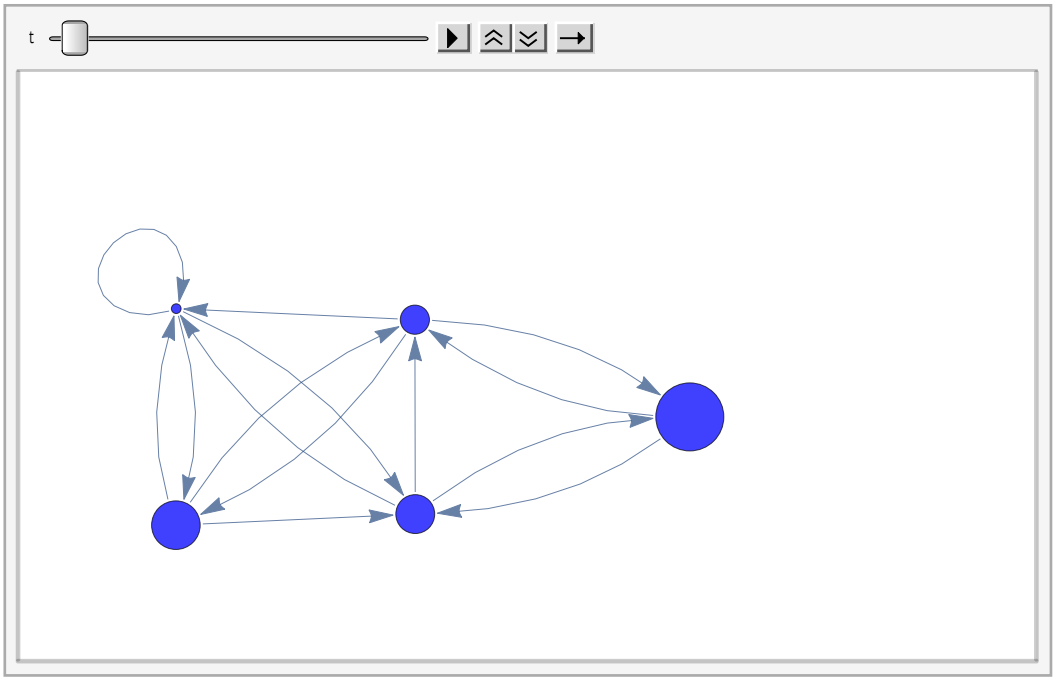
In[12]:=
```
u = {0.05, 0.2, 0.35, 0.15, 0.25};
```

Random walk on C_5

K_1

In[14]:=
```
Animate[MarkovDistributionPlot[K1, u.MatrixPower[K1, t]], {t, 0, 100, 1},
 AnimationRunning → False, AnimationRepetitions → 1, AnimationRate → 4]
```

Out[14]=



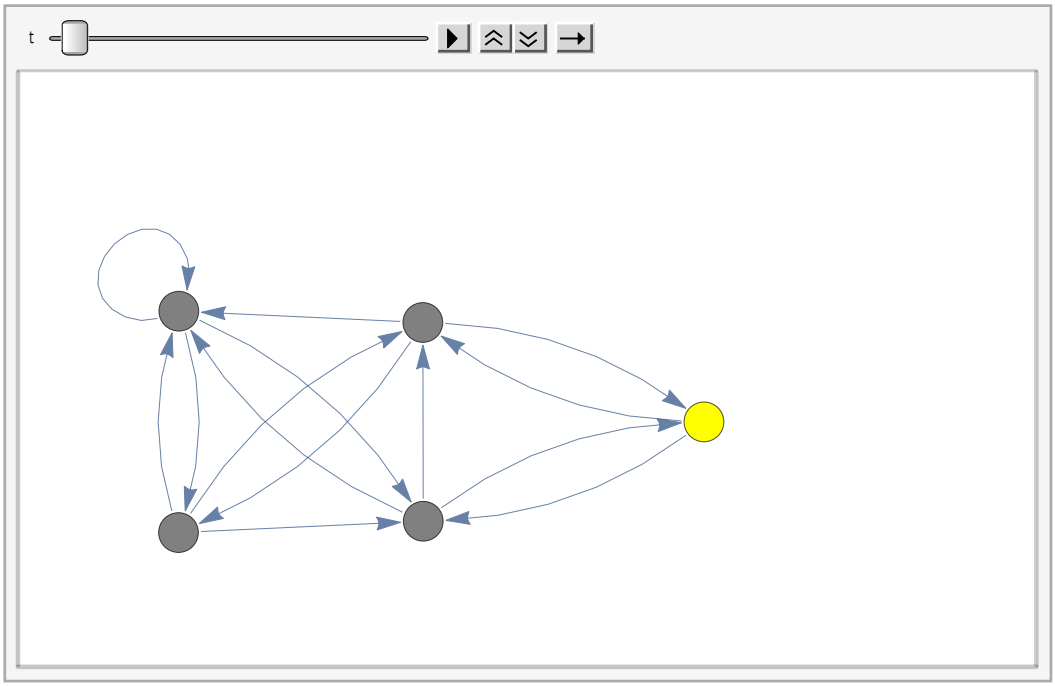
Evolution of the state

Random walk on C_5

K_5

In[19]:=
```
X0 = 3;  (* Initial state *)
T = 100;  (* Number of time steps *)
X = MarkovChain[K1, X0, T];
Animate[MarkovActivePlot[K1, X[[t + 1]]], {t, 0, T - 1, 1},
 AnimationRunning → False, AnimationRepetitions → 1, AnimationRate → 5]
```

Out[22]=



Evolution of empirical distribution

Random walk on C_5

K_5

In[30]:=
```
u0 = {1, 0, 0, 0, 0};  (* Distribution of the initial state *)
T = 100;  (* Number of time steps *)
n = 1000;  (* Number of independent samples *)
XS0 = RandomChoice[u0 → Range[Length[u0]], n];  (* n independent samples from u0 *)
XS = Table[MarkovChain[K1, XS0[[k]], T], {k, 1, n}];
h = Table[Normalize[
    BinCounts[Transpose[XS][[t + 1]], {0.5, Length[u0] + 0.5, 1}], Total], {t, 0, T - 1}];
(* Empirical distribution of n independent runs in T consecutive steps *)
Animate[MarkovDistributionPlot[K1, h[[t + 1]]], {t, 0, T - 1, 1},
 AnimationRunning → False, AnimationRepetitions → 1, AnimationRate → 4]
```

Out[36]=