

Full Name: _____

Student No: _____

Attempt: ☐ First ☐ SecondGrade:

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Read before you start:

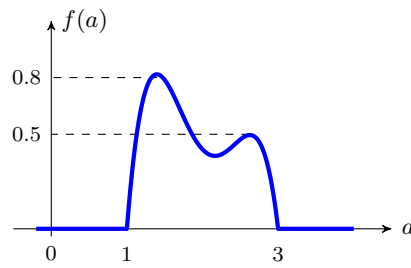
- Please make sure you write your full name and student number on everything you hand in.
- To avoid chaos, please make sure to exactly follow the proctors' instructions.
- The exam has **two parts** and **four phases**:
 - ➡ Phase 1 (20 minutes): You work individually on **Part A** and submit your solutions.
 - ➡ Phase 2 (60 minutes): You work individually on **Part B** and submit your solutions.
 - ➡ Phase 3 (20 minutes): You discuss the problems in **Part B** with other students.
 - ➡ Phase 4 (30 minutes): You work individually and submit updated solutions to all or some of the problems in **Part B**.
- Phases 3 and 4 are optional. You can opt to skip them and use their time to continue working individually on **Part B**.
- Important: In order to participate in Phases 3 and 4, you *must* first submit your first attempt on **Part B**.
- **All answers require justifications.**

You can use the remainder of this page as scratch paper.

Part B

3. (10 points) If A and B are two independent standard normal random variables, what is the distribution of $3A - 4B + 1$?
4. (10 points) Let U be a random variable uniformly distributed over the interval $[0, 1]$ and let $X := -\ln(U)$.
- (a) What are the possible values of X ?
- (b) Find the probability density function of X .

5. (15 points) The following depicts the probability density function of a continuous random variable X :



(a) What are the possible values of X ?

(b) What is the probability that $-2X + 3 \leq 1$?

(c) Sketch the probability density function of $Y = -2X + 3$.

6. (10 points) In a quiet afternoon, two shopkeepers, Laila and Jad, are waiting for customers to arrive at a shop. Being bored, they decide to make a bet on the arrival time of the first customer. They agree that
- If the first customer arrives within the next half an hour, Jad pays 2 dollars to Laila.
 - If not, Laila pays $T - 0.5$ dollars to Jad, where T is arrival time of the first customer in hours.

For instance, if the first customer arrives 15 minutes after the bet is made, Laila wins 2 dollars, but if the first customer arrives 45 minutes after the bet is made, Jad wins $0.75 - 0.5$ dollars.

We assume that T has an exponential distribution with rate $\lambda = 1$ per hour.

Recall: The cdf of an exponential random variable with rate λ is $F(t) = 1 - e^{-\lambda t}$ for $t \geq 0$ and $F(t) = 0$ for $t < 0$.

(a) What is the probability that Jad wins more than 1 dollar?

(b) Is this game fair? If not, in whose favour?

7. (10 points) A bag contains 5 balls: 3 blue, 1 green, and 1 red. You repeat drawing balls from the bag at random without replacement until the red ball is drawn, after which you stop. For each blue ball that you draw, you are paid 1 dollar and if you draw the green ball, you are paid an additional 2 dollars.

(a) What is the probability that, during the game, the green ball is drawn?

(b) Find your expected gain.

You can use this page as extra space for your solutions.