Full Name: _____

Student No: _____

Attempt: ☐ First ☐ Second

Grade:

Read before you start:

- The exam has **two parts** and **three phases**:
  - ♣ Phase 1 (70 minutes): You work individually on **both parts** and submit your solutions.
  - ♣ Phase 2 (20 minutes): You discuss the problems in **part 2** in pairs or groups of three.
  - ♣ Phase 3 (30 minutes): You work individually and submit updated solutions to all or some of the problems in **part 2**.

  Phases 2 and 3 are optional.
- Please make sure you write your full name and student number on everything you hand in.
- To avoid chaos, please make sure to exactly follow the proctors' instructions.
- **All answers require justifications**.

You can use the remainder of this page as scratch paper.

Part 2

1. (15 points) We roll two ordinary dice and record the numbers shown on them.

   (a) Identify the sample space and the probability of each possible outcome.

   (b) What is the probability that the sum of the two numbers is odd?

   (c) What is the probability that the product of the two numbers is odd?

2. (10 points) Four black socks, six white socks, and three blue socks are in a drawer.

   (a) What is the probability that two randomly picked socks are of the same colour?

   (b) What is the probability that among three randomly picked socks, there are at least two with the same colour?

3. (20 points) A jar contains 5 blue balls, 3 red balls, and 7 yellow balls. We repeat drawing balls from the jar at random with replacement until a red ball is drawn.

   (a) What is the probability that the red ball is drawn on the 5th pick?

   (b) What is the probability that, during the experiment, we draw exactly 3 blue balls and 4 yellow balls.

   (c) What is the probability that, during the experiment, we draw at least one blue ball?

   (d) (bonus) What is the probability that, during this experiment, we draw at least two blue balls?
   Hint: A bit of reasoning may save you from an intimidating computation.

4. (15 points) In a digital communication channel, messages are sent and received as strings of 0s and 1s. Due to noise, the transmission of each bit of information is subject to error (i.e., receiving a 0 instead of a 1, or vice versa).

   Laila and Jad are communicating through a rather unreliable digital communication channel in which every bit is transmitted incorrectly with probability 0.1. Using this channel, Laila wants to communicate whether or not she will be available for a date on Friday evening. To make her transmission more reliable, she repeats her answer three times. In other words, if her answer is "yes", she sends 111 and if her answer is "no", she sends 000. We assume that Laila's answer is "yes" with probability $2/5$ and "no" with probability $3/5$.

   (a) Propose a reasonable strategy for Jad to recover Laila's message from what he receives through the channel.

   (b) What is the probability that Jad receives 110?

   (c) Suppose that Jad has received 110. What is the probability that Laila's answer has been "yes"?

You can use this sheet as extra space for your solutions.

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