American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

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Chapter 5
Discrete random variables
Part 3

Discrete random variables

Discrete random variables

Some important types of discrete RVs:

- Bernoulli RVs (we will discuss if time permits)
- Binomial RVs
- Poisson RVs
- Geometric RVs (not covered in this course)
- Hypergeometric RVs (not covered in this course)
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Poisson RVs

Example (Radioactive emission)

A block of radioactive material emits γ-particles at the average rate of 30 per minute. Researchers working on a type of radiation therapy, are interested in the number of γ-particles emitted during a 30-second period.

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Q3 What is the standard deviation of N?

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Example (Visits to a website)

A popular website receives, on average, 30 unique visitors per hour. Due to a technical issue, the network administrator will be performing a server maintenance from 15:00 till 15:30, during which the website will be down. The administrator is interested in the number of people trying to access the website during this period.

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Do you see a resemblance with the previous example?

Poisson RVs

Consider a certain type of incidents (or occurrences) happening over time in such a way that

- The incidents occur at random with a certain average rate.
- At any moment in time, the occurrence time of the next incident is independent of the previous incidents.

Poisson (an 18th/19th century mathematician) argued that the number N of such incidents occurring during a time interval I approximately has a Poisson distribution:

- The possible values of N are $0, 1, 2, \ldots$.
- For each possible value k,

$$\boxed{\mathbb{P}(N = k) = \mathrm{e}^{-\lambda} \frac{\lambda^k}{k!}}$$

where the parameter λ is the mean # of incidents during I.

A discrete RV with the Poisson distribution is called a Poisson RV.

Poisson RVs

In the last two examples, each of the two RVs

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A1 The average rate of particle emission is 30 per minute. Therefore, the mean number of particles within 30 seconds (half a minute) is $\boxed{\lambda = 15}$.

A2 The average rate of unique visits to the website is 30 per hour. Therefore, the mean number of unique visitors within half an hour is $\boxed{\lambda = 15}$.

Poisson RVs

In general, if N is a Poisson RV with parameter λ, then

- The expected value of N is

$$\boxed{\mathbb{E}[N] = \lambda}$$

(This is consistent with the interpretation!)

- The variance of N is

$$\boxed{\sigma^2 = \mathbb{V}\mathrm{ar}[N] = \lambda}$$

hence the standard deviation of N is

$$\boxed{\sigma = \mathbb{SD}[N] = \sqrt{\lambda}}$$

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A We have

$$\mathbb{P}(N = 3) = \mathrm{e}^{-5} \times \frac{5^3}{3!} \approx 0.006738 \times \frac{125}{3 \times 2 \times 1} \approx \boxed{0.1404}$$

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A $\mathbb{SD}[N] = \sqrt{5} \approx \boxed{2.2361}$.

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Poisson RVs

Let us do find the same values using the following applet:

https://homepage.divms.uiowa.edu/~mbognar/

Statistical software such as R:

https://www.r-project.org/

have functions for computing Poisson and other common distributions. (Why don't you try this on R?)

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