American University of Beirut

STAT 210: Elementary Statistics for Sciences

2022–2023 Fall

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Chapter 5

Discrete random variables

Part 2

Discrete random variables

Discrete random variables

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$

$$\boxed{\mathbb{P}(\mathtt{HH}) = 0.3025 \qquad \mathbb{P}(\mathtt{HT}) = 0.2475 \qquad \mathbb{P}(\mathtt{TH}) = 0.2475 \qquad \mathbb{P}(\mathtt{TT}) = 0.2025}$$

$N := \#$ of heads

outcome	HH	HT	TH	TT
N	2	1	1	0

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$N := \#$ of heads

outcome	HH	HT	TH	TT
N	2	1	1	0

Q What is the mean value of N?

A1 We have

$$\begin{aligned}\mathbb{E}[N] &= \mathbb{P}(\mathtt{HH}) \times 2 + \mathbb{P}(\mathtt{HT}) \times 1 + \mathbb{P}(\mathtt{TH}) \times 1 + \mathbb{P}(\mathtt{TT}) \times 0 \\ &= 0.3025 \times 2 + 0.2475 \times 1 + 0.2475 \times 1 + 0.2025 \times 0 \\ &= \boxed{1.1}\end{aligned}$$

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$N := \#$ of heads

outcome	HH	HT	TH	TT
N	2	1	1	0

Q What is the mean value of N?

A2 Using the distribution of N, we have

$$\begin{aligned}
\mathbb{E}[N] &= \mathbb{P}(N=0) \times 0 + \mathbb{P}(N=1) \times 1 + \mathbb{P}(N=2) \times 2 \\
&= 0.2025 \times 0 + 0.4950 \times 1 + 0.3025 \times 2 \\
&= \boxed{1.1}
\end{aligned}$$

k	$\mathbb{P}(N=k)$
0	0.2025
1	0.4950
2	0.3025

Discrete random variables

Example (Rolling two dice)

$$\boxed{\Omega := \{⚀⚀, ⚀⚁, ⚀⚂, \ldots, ⚅⚅\}}$$

$$\boxed{\mathbb{P}(⚀⚀) = \mathbb{P}(⚀⚁) = \mathbb{P}(⚀⚂) = \cdots = \mathbb{P}(⚅⚅) = 1/36}$$

$X :=$ sum of the numbers on the dice

Discrete random variables

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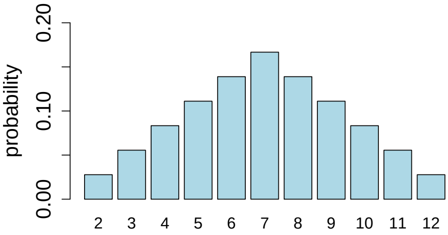
Q What is the mean value of X?

A1 Since the distribution of X is symmetric around 7, $\mathbb{E}[X] = \boxed{7}$.



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Q What is the mean value of X?

A2 We have

$$\mathbb{E}[X] = \mathbb{P}(⚀⚀) \times 2 + \mathbb{P}(⚀⚁) \times 3 + \mathbb{P}(⚀⚂) \times 4 + \cdots + \mathbb{P}(⚅⚅) \times 12$$

$$= \frac{\overbrace{2 + 3 + 4 + \cdots + 12}^{\text{values for each of the 36 possible outcomes}}}{36}$$

$$= \frac{2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 6 \times 7 + 5 \times 8 + 4 \times 9 + 3 \times 10 + 2 \times 11 + 12}{36}$$

$$= \boxed{7}$$

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$X :=$ sum of the numbers on the dice

Q What is the mean value of X?

A3 Using the distribution of X, we have

$$\begin{aligned}\mathbb{E}[X] &= \mathbb{P}(X=2)\times 2 + \mathbb{P}(X=3)\times 3 + \mathbb{P}(X=4)\times 4 + \cdots \\ &\qquad + \mathbb{P}(X=12)\times 12 \\ &= \frac{1}{36}\times 2 + \frac{2}{36}\times 3 + \frac{3}{36}\times 4 + \cdots + \frac{1}{36}\times 12 \\ &= \boxed{7}\end{aligned}$$

a	$\mathbb{P}(X=a)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

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Q What is the mean value of X?

A4 Note that

- The mean of the number on the 1st die is 3.5.
- The mean of the number on the 2nd die is 3.5.

Therefore,

- The mean of the sum of the two numbers is $3.5 + 3.5 = \boxed{7}$.

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$X :=$ sum of the numbers on the dice

Q What is the **mean** value of X?

$\boxed{\text{A4}}$ More precisely,

$$X = X_1 + X_2$$

$X_1 :=$ number on the 1st die

$X_2 :=$ number on the 2nd die

Now, $\mathbb{E}[X_1] = \mathbb{E}[X_2] = 3.5$, hence

$$\mathbb{E}[X] = \mathbb{E}[X_1] + \mathbb{E}[X_2] = 3.5 + 3.5 = \boxed{7}$$

Expected value of discrete RVs

The expected value (a.k.a. the mean) of a discrete RV X is the average value of X over all the possible outcomes of the experiment weighted by their probabilities, and is denoted by $\mathbb{E}[X]$ or μ.

It is interpreted as the (approximate) mean of the values of X in many many repeated experiments.

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1. If the experiment has possible outcomes $\omega_1, \omega_2, \omega_3, \ldots$, then

$$\boxed{\mathbb{E}[X] = \mathbb{P}(\omega_1) \cdot X(\omega_1) + \mathbb{P}(\omega_2) \cdot X(\omega_2) + \mathbb{P}(\omega_3) \cdot X(\omega_3) + \cdots}$$

[similar to using the raw data]

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2. If X has possible values $a_1, a_2, a_3, \ldots$, then

$$\boxed{\mathbb{E}[X] = \mathbb{P}(X = a_1) \cdot a_1 + \mathbb{P}(X = a_2) \cdot a_2 + \mathbb{P}(X = a_3) \cdot a_3 + \cdots}$$

[similar to using the relative frequency table]

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If the experiment involves picking a member of a population uniformly at random (i.e., all members being equally likely to be picked), then the expected value of a RV X is the same as its population mean.

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Example

The expected weight of a bird chosen uniformly at random from the population of all birds within a species is the same as the population mean of the weight for that species.

Standard deviation of discrete RVs

The variance of a discrete RV X is the expected value of the square of the deviation of X from its mean, and is denoted by $\mathbb{V}\mathrm{ar}[X]$ or σ^2.

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As before, the expected values of $(X-\mu)^2$ and X^2 can be calculated in two different ways, either by summing over the possible outcomes of the experiment, or by summing over the possible values of X.

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Binomial RVs

Example (Gene with two alleles)

A certain type of gene in humans has two alleles (versions) A and B. Previous studies suggest that around 70% of the people have allele A and the remaining 30% have allele B. A random sample of 10 individuals are selected.

$$X := \# \text{ of sampled individuals with allele } A$$

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Binomial RVs

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A coin has a bias in favor of heads:

- 70% of the times the coin comes up heads.
- 30% of the times the coin comes up tails.

We flip the coin 10 times in a row.

$$X := \# \text{ of heads}$$

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Do you notice something familiar??!

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Consider an experiment involving n trials (or subexperiments), each with two possible results: success or failure. Suppose that

- Each trial is successful with probability p.
- The trials are independent of one another.

The random variable

$$X := \# \text{ of successes}$$

is called a binomial RV with parameters n and p.

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Clearly, in the last two examples, the two RVs

- $X := \#$ of sampled individuals with allele A
- $X := \#$ of heads

are both binomial RVs with parameters $n = 10$ and $p = 0.7$.

Binomial RVs

Suppose X is a binomial RV with parameters $n = 10$ and $p = 0.7$.

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The same kind of computation/visualization (and much more) can be done using statistical software such as R:

https://www.r-project.org/

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If k is a possible value, then

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Binomial coefficient

The notation $\binom{n}{k}$ (or ${}_nC_k$) stands for the number of ways one can select k distinct objects from a collection of n distinguishable objects, disregarding the order of the selection.

It can be calculated as

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

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e.g.,

$$\mathbb{P}(X = 7) = \binom{10}{7}(0.7)^7(0.3)^3 \approx \boxed{0.2668279}$$

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Binomial RVs

Suppose X is a binomial RV with parameters $n = 10$ and $p = 0.7$.

Q3 What is the standard deviation of X?

A We have

$$\sigma^2 = \mathbb{V}\mathrm{ar}[X] = 10 \times 0.7 \times 0.3 = \boxed{2.1}$$

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Hence, the standard deviation is

$$\sigma = \mathbb{SD}[X] = \sqrt{\mathbb{V}\text{ar}[X]} = \sqrt{2.1} \approx \boxed{1.449138}$$

Binomial RVs

Example (Gene with two alleles)

A certain type of gene in humans has two alleles (versions) A and B. Previous studies suggest that around 70% of the people have allele A and the remaining 30% have allele B. A random sample of 10 individuals are selected.

$$X := \# \text{ of sampled individuals with allele } A$$

Q1 What is the distribution of X? (e.g., what is $\mathbb{P}(X = 7)$?)

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Binomial RVs

In general, if X is a binomial RV with parameters n and p, then

- The possible values of X are $0, 1, 2, \ldots, n$.
  If k is a possible value, then

$$\boxed{\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^k}$$

- The <u>expected value</u> of X is

$$\boxed{\mathbb{E}[X] = np}$$

- The <u>variance</u> of X is

$$\boxed{\sigma^2 = \mathbb{V}\mathrm{ar}[X] = np(1-p)}$$

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$$\mathbb{P}(X \leq 1) = \mathbb{P}(X = 0) + \mathbb{P}(X = 1)$$

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$$\begin{aligned}\mathbb{P}(X \leq 1) &= \mathbb{P}(X = 0) + \mathbb{P}(X = 1) \\ &= \binom{10}{0}(0.7)^0(0.3)^{10} + \binom{10}{1}(0.7)^1(0.3)^9\end{aligned}$$

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&= 0.0000059049 + 0.000137781
\end{aligned}$$

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&= 0.0000059049 + 0.000137781 \\
&= 0.0001436859 \approx \boxed{1.44 \times 10^{-4}}
\end{aligned}$$

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A1 We have

$$\mathbb{P}(X \leq 8)$$

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Q5 What is the probability that at most 8 individual have A?

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$$\mathbb{P}(X \leq 8) = \mathbb{P}(X = 0) + \mathbb{P}(X = 1) + \cdots + \mathbb{P}(X = 8)$$

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$$\begin{aligned}\mathbb{P}(X \leq 8) &= \mathbb{P}(X = 0) + \mathbb{P}(X = 1) + \cdots + \mathbb{P}(X = 8) \\ &= \binom{10}{0}(0.7)^0(0.3)^{10} + \binom{10}{1}(0.7)^1(0.3)^9 + \cdots + \binom{10}{8}(0.7)^8(0.3)^2\end{aligned}$$

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&= \ldots \qquad \text{(Ugh! Too much calculation!)}
\end{aligned}$$

Binomial RVs

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$$X := \# \text{ of sampled individuals with allele } A$$

Q5 What is the probability that at most 8 individual have A?

A2 We have

$$\mathbb{P}(X \leq 8)$$

Binomial RVs

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A certain type of gene in humans has two alleles (versions) A and B. Previous studies suggest that around 70% of the people have allele A and the remaining 30% have allele B. A random sample of 10 individuals are selected.

$$X := \# \text{ of sampled individuals with allele } A$$

Q5 What is the probability that at most 8 individual have A?

A2 We have

$$\mathbb{P}(X \leq 8) = 1 - \mathbb{P}(X = 9) - \mathbb{P}(X = 10)$$

Binomial RVs

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A certain type of gene in humans has two alleles (versions) A and B. Previous studies suggest that around 70% of the people have allele A and the remaining 30% have allele B. A random sample of 10 individuals are selected.

$$X := \# \text{ of sampled individuals with allele } A$$

Q5 What is the probability that at most 8 individual have A?

$\boxed{\text{A2}}$ We have

$$\begin{aligned}\mathbb{P}(X \leq 8) &= 1 - \mathbb{P}(X = 9) - \mathbb{P}(X = 10) \\ &= 1 - \binom{10}{9}(0.7)^9(0.3)^1 - \binom{10}{10}(0.7)^{10}(0.3)^0\end{aligned}$$

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A certain type of gene in humans has two alleles (versions) A and B. Previous studies suggest that around 70% of the people have allele A and the remaining 30% have allele B. A random sample of 10 individuals are selected.

$$X := \# \text{ of sampled individuals with allele } A$$

Q5 What is the probability that at most 8 individual have A?

A2 We have

$$\begin{aligned}
\mathbb{P}(X \leq 8) &= 1 - \mathbb{P}(X = 9) - \mathbb{P}(X = 10) \\
&= 1 - \binom{10}{9}(0.7)^9(0.3)^1 - \binom{10}{10}(0.7)^{10}(0.3)^0 \\
&= 1 - 0.121060821 - 0.0282475249
\end{aligned}$$

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&= 1 - \binom{10}{9}(0.7)^9(0.3)^1 - \binom{10}{10}(0.7)^{10}(0.3)^0 \\
&= 1 - 0.121060821 - 0.0282475249 \\
&\approx \boxed{0.850692}
\end{aligned}$$