American University of Beirut
STAT 210: Elementary Statistics for Sciences
2022–2023 Fall

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Chapter 4
Probability
Part 1

Probability

Language of probabilities

Reasoning about any matter (scientifically) requires simple, precise and unambiguous usage of language.

The language of probabilities is developed to help us reason about chance and randomness.

It is based on the concept of a probability model.

Statistical analysis is (almost always) performed in the language of probabilities, and using probability models.

Random experiments

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- Flip a coin twice.

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Examples

- Flip a coin.
- Roll an (ordinary 6-sided) die.
- Flip a coin twice.
- Randomly pick two distinct students from this class.

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Examples

- Flip a coin.
- Roll an (ordinary 6-sided) die.
- Flip a coin twice.
- Randomly pick two distinct students from this class.
- Tomorrow's weather condition.

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- Tomorrow's weather condition.
- The next parliamentary election.

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A random experiment refers to any specific experiment (or phenomenon) whose outcome is a matter of chance (i.e., determined at random).

Examples

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- Roll an (ordinary 6-sided) die.
- Flip a coin twice.
- Randomly pick two distinct students from this class.
- Tomorrow's weather condition.
- The next parliamentary election.

Question (food for thought)

What does it mean to say the outcome is a matter of chance?

Probability model of a random experiment

Example (Flipping a coin)

Consider the random experiment of flipping a (fair) coin.

Probability model of a random experiment

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The experiment has two possible outcomes:

- It comes up heads.
- It comes up tails.

Probability model of a random experiment

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The collection of all possible outcomes for the experiment

$$\Omega := \{\text{heads}, \text{tails}\}$$

is called the sample space for this experiment.

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The two outcomes are equally likely. We express this by writing

$$\begin{aligned} \mathbb{P}(\text{heads}) &= 1/2 \\ \mathbb{P}(\text{tails}) &= 1/2 \end{aligned}$$

Probability model of a random experiment

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The collection of all possible outcomes for the experiment

$$\Omega := \{\mathsf{heads}, \mathsf{tails}\}$$

is called the sample space for this experiment.

The two outcomes are equally likely. We express this by writing

$$\mathbb{P}(\mathsf{heads}) = 1/2 \qquad \text{(read: "The probability of heads is } 1/2\text{.")}$$

$$\mathbb{P}(\mathsf{tails}) = 1/2 \qquad \text{(read: "The probability of tails is } 1/2\text{.")}$$

Probability model of a random experiment

Example (Flipping a biased coin)

Consider the random experiment of flipping a biased coin.

Probability model of a random experiment

Example (Flipping a biased coin)

Consider the random experiment of flipping a biased coin.

The sample space (collection of all possible outcomes) is again

$$\Omega := \{\mathtt{H}, \mathtt{T}\}$$

Probability model of a random experiment

Example (Flipping a biased coin)

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Suppose the coin has a bias in favor of heads:

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Probability model of a random experiment

Example (Flipping a biased coin)

Consider the random experiment of flipping a biased coin.

The sample space (collection of all possible outcomes) is again

$$\Omega := \{\mathtt{H}, \mathtt{T}\}$$

Suppose the coin has a bias in favor of heads:

- 55% of the times the coin comes up heads.
- 45% of the times the coin comes up tails.

We express this by writing

$$\mathbb{P}(\mathtt{H}) = 0.55 \qquad \mathbb{P}(\mathtt{T}) = 0.45$$

Probability model of a random experiment

Example (Rolling a die)

Consider the random experiment of rolling an (ordinary) die.

Probability model of a random experiment

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Consider the random experiment of rolling an (ordinary) die.

The experiment has six possible outcomes:

⚀ ⚁ ⚂ ⚃ ⚄ ⚅

Probability model of a random experiment

Example (Rolling a die)

Consider the random experiment of rolling an (ordinary) die.

The experiment has six possible outcomes:

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The sample space (collection of all possible outcomes) is

$$\Omega := \{\text{⚀}, \text{⚁}, \text{⚂}, \text{⚃}, \text{⚄}, \text{⚅}\} \; .$$

Probability model of a random experiment

Example (Rolling a die)

Consider the random experiment of rolling an (ordinary) die.

The experiment has six possible outcomes:

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$$\Omega := \{\text{⚀}, \text{⚁}, \text{⚂}, \text{⚃}, \text{⚄}, \text{⚅}\} \ .$$

The possible outcomes are all equally likely. We write

$$\mathbb{P}(\text{⚀}) = \mathbb{P}(\text{⚁}) = \mathbb{P}(\text{⚂}) = \mathbb{P}(\text{⚃}) = \mathbb{P}(\text{⚄}) = \mathbb{P}(\text{⚅}) = {}^{1}\!/_{6} \ .$$

Probability model of a random experiment

Example (Rolling a die)

$$\boxed{\Omega := \{\text{⚀}, \text{⚁}, \text{⚂}, \text{⚃}, \text{⚄}, \text{⚅}\}}$$

$$\boxed{\mathbb{P}(\text{⚀}) = \mathbb{P}(\text{⚁}) = \mathbb{P}(\text{⚂}) = \mathbb{P}(\text{⚃}) = \mathbb{P}(\text{⚄}) = \mathbb{P}(\text{⚅}) = {}^{1}\!/_{6}}$$

Probability model of a random experiment

Example (Rolling a die)

$$\boxed{\Omega := \{\epsdice{1}, \epsdice{2}, \epsdice{3}, \epsdice{4}, \epsdice{5}, \epsdice{6}\}}$$

$$\boxed{\mathbb{P}(\epsdice{1}) = \mathbb{P}(\epsdice{2}) = \mathbb{P}(\epsdice{3}) = \mathbb{P}(\epsdice{4}) = \mathbb{P}(\epsdice{5}) = \mathbb{P}(\epsdice{6}) = 1/6}$$

Q What is the chance that the die shows an even number?

Probability model of a random experiment

Example (Rolling a die)

$$\boxed{\Omega := \{\epsdice{1}, \epsdice{2}, \epsdice{3}, \epsdice{4}, \epsdice{5}, \epsdice{6}\}}$$

$$\boxed{\mathbb{P}(\epsdice{1}) = \mathbb{P}(\epsdice{2}) = \mathbb{P}(\epsdice{3}) = \mathbb{P}(\epsdice{4}) = \mathbb{P}(\epsdice{5}) = \mathbb{P}(\epsdice{6}) = {}^1\!/_6}$$

Q What is the chance that the die shows an even number?

$\boxed{\text{A1}}$ ${}^1\!/_2$ (by symmetry).

Probability model of a random experiment

Example (Rolling a die)

$$\boxed{\Omega := \{\epsdice{1}, \epsdice{2}, \epsdice{3}, \epsdice{4}, \epsdice{5}, \epsdice{6}\}}$$

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Q What is the chance that the die shows an even number?

A2 The event that the die shows an even number is identified by the collection of outcomes that realize it:

$$\langle \text{die shows an even number} \rangle = \{\epsdice{2}, \epsdice{4}, \epsdice{6}\}$$

Probability model of a random experiment

Example (Rolling a die)

$$\boxed{\Omega := \{\epsdice{1}, \epsdice{2}, \epsdice{3}, \epsdice{4}, \epsdice{5}, \epsdice{6}\}}$$

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Q What is the chance that the die shows an even number?

A2 The event that the die shows an even number is identified by the collection of outcomes that realize it:

$$\langle \text{die shows an even number} \rangle = \{\epsdice{2}, \epsdice{4}, \epsdice{6}\}$$

Its probability is the sum of the probabilities of the realizing outcomes:

$$\begin{aligned} \mathbb{P}(\text{die shows an even number}) &= \mathbb{P}(\epsdice{2}) + \mathbb{P}(\epsdice{4}) + \mathbb{P}(\epsdice{6}) \\ &= {}^1\!/_6 + {}^1\!/_6 + {}^1\!/_6 = \boxed{{}^1\!/_2}\,. \end{aligned}$$

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Probability model of a random experiment

Example (Flipping a fair coin twice)

Consider the random experiment of flipping a fair coin twice.

Q What is the sample space?

$\boxed{\text{A}}$ $\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}$.

Probability model of a random experiment

Example (Flipping a fair coin twice)

Consider the random experiment of flipping a fair coin twice.

Q What is the sample space?

A $\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}$.

Q What is the probability of each outcome?

Probability model of a random experiment

Example (Flipping a fair coin twice)

Consider the random experiment of flipping a fair coin twice.

Q What is the sample space?

A $\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}$.

Q What is the probability of each outcome?

A1 By symmetry, the outcomes are all equally likely, hence

$$\mathbb{P}(\mathtt{HH}) = \mathbb{P}(\mathtt{HH}) = \mathbb{P}(\mathtt{HH}) = \mathbb{P}(\mathtt{HH}) = 1/4 \ .$$

Probability model of a random experiment

Example (Flipping a fair coin twice)

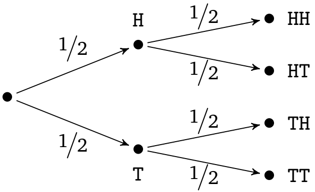
Consider the random experiment of flipping a fair coin twice.

Q What is the sample space?

A $\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}$.

Q What is the probability of each outcome?

A2 Consider the tree of possibilities.



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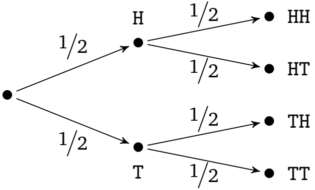


$$\mathbb{P}(\mathtt{HH}) = 1/2 \times 1/2 = 1/4$$

Probability model of a random experiment

Example (Flipping a fair coin twice)

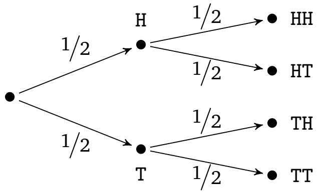
Consider the random experiment of flipping a fair coin twice.

(Q) What is the sample space?

$\boxed{\text{A}}$ $\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}$.

(Q) What is the probability of each outcome?

$\boxed{\text{A2}}$ Consider the tree of possibilities.



$$\mathbb{P}(\mathtt{HH}) = 1/2 \times 1/2 = 1/4$$

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Probability model of a random experiment

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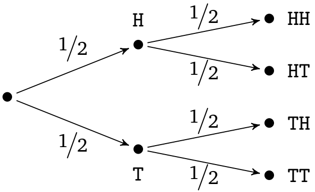

$$\mathbb{P}(\mathtt{HH}) = 1/2 \times 1/2 = 1/4$$
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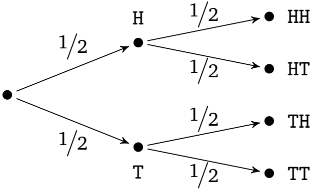

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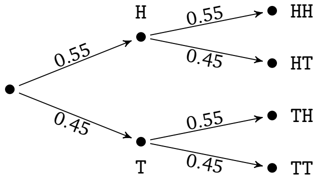

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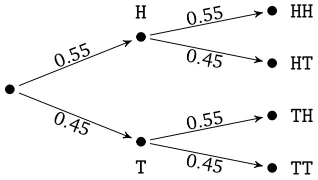

$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$

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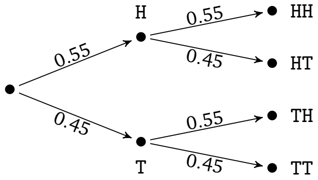

$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
$$\mathbb{P}(\mathtt{HT}) = 0.55 \times 0.45 = 0.2475$$

Probability model of a random experiment

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Q What is the probability of each outcome?

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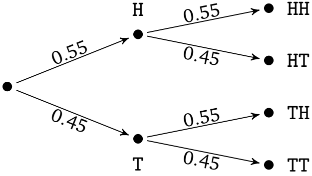

$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
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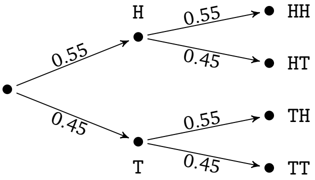

$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
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Probability model of a random experiment

Example (Flipping a biased coin twice)

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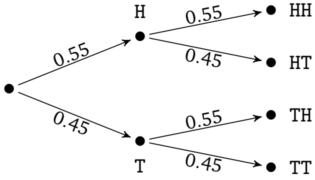

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$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Probability model of a random experiment

Example (Flipping a biased coin twice)

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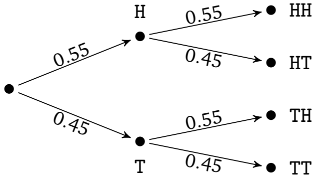

$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
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$$\mathbb{P}(\mathtt{TH}) = 0.45 \times 0.55 = 0.2475$$
$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Note that

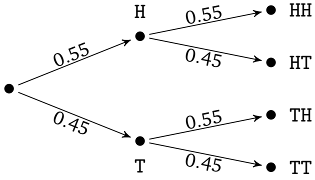
$$\mathbb{P}(\mathtt{HH}) + \mathbb{P}(\mathtt{HT}) + \mathbb{P}(\mathtt{TH}) + \mathbb{P}(\mathtt{TT}) = 0.3025 + 0.2475 + 0.2475 + 0.2025 = 1$$

as expected.

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



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$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Note that

$$\mathbb{P}(\mathtt{HH}) + \mathbb{P}(\mathtt{HT}) + \mathbb{P}(\mathtt{TH}) + \mathbb{P}(\mathtt{TT}) = 0.3025 + 0.2475 + 0.2475 + 0.2025 = 1$$

as expected.

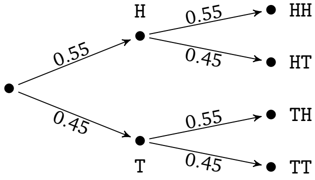
In general:

The probabilities of individual outcomes always add up to 1.

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
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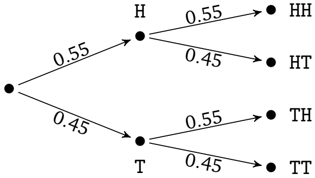
$$\mathbb{P}(\mathtt{TH}) = 0.45 \times 0.55 = 0.2475$$
$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Q What is the probability that <u>both flips show the same side</u>?

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
$$\mathbb{P}(\mathtt{HT}) = 0.55 \times 0.45 = 0.2475$$
$$\mathbb{P}(\mathtt{TH}) = 0.45 \times 0.55 = 0.2475$$
$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Q What is the probability that <u>both flips show the same side</u>?

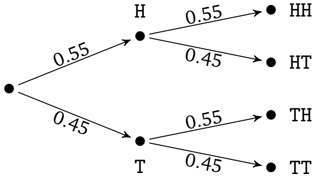
$\boxed{\text{A}}$ This event is realized by two possible outcomes:

$$\langle \text{both flips show same side} \rangle = \{\mathtt{HH}, \mathtt{TT}\}$$

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



$$\mathbb{P}(\mathtt{HH}) = 0.55 \times 0.55 = 0.3025$$
$$\mathbb{P}(\mathtt{HT}) = 0.55 \times 0.45 = 0.2475$$
$$\mathbb{P}(\mathtt{TH}) = 0.45 \times 0.55 = 0.2475$$
$$\mathbb{P}(\mathtt{TT}) = 0.45 \times 0.45 = 0.2025$$

Q What is the probability that <u>both flips show the same side</u>?

$\boxed{\text{A}}$ This event is realized by two possible outcomes:

$$\langle \text{both flips show same side} \rangle = \{\mathtt{HH}, \mathtt{TT}\}$$

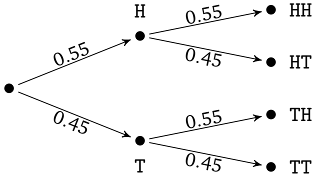
Therefore,

$$\mathbb{P}(\text{both flips show same side}) = \mathbb{P}(\mathtt{HH}) + \mathbb{P}(\mathtt{TT})$$

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



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$$\langle \text{both flips show same side} \rangle = \{\mathtt{HH}, \mathtt{TT}\}$$

Therefore,

$$\begin{aligned} \mathbb{P}(\text{both flips show same side}) &= \mathbb{P}(\mathtt{HH}) + \mathbb{P}(\mathtt{TT}) \\ &= 0.3025 + 0.2025 = \boxed{0.5050} \end{aligned}$$

Probability model of a random experiment

Example (Flipping a biased coin twice)

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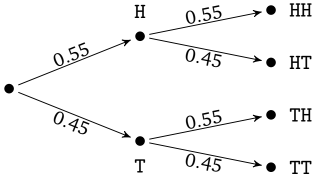

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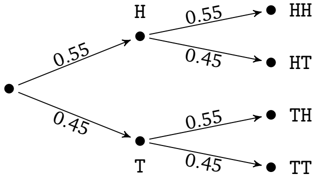
Q What is the probability that the <u>2nd flips comes up heads</u>?

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



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Q What is the probability that the 2nd flips comes up heads?

$\boxed{\text{A1}}$ Obviously 0.55. We already know the coin comes up heads 55% of the times.

Probability model of a random experiment

Example (Flipping a biased coin twice)

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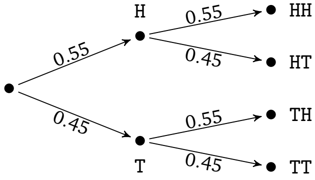

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Q What is the probability that the 2nd flips comes up heads?

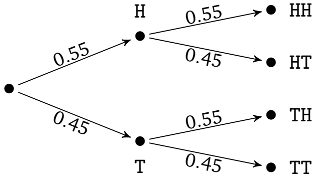
A2 This event is realized by two possible outcomes:

$$\langle \text{2nd flip comes up heads} \rangle = \{\mathtt{HH}, \mathtt{TH}\}$$

Probability model of a random experiment

Example (Flipping a biased coin twice)

$$\boxed{\Omega := \{\mathtt{HH}, \mathtt{HT}, \mathtt{TH}, \mathtt{TT}\}}$$



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Probability model of a random experiment

Example (Flipping a biased coin twice)

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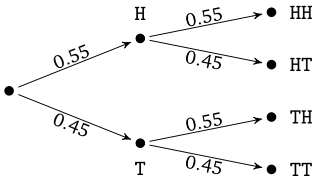

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A2 This event is realized by two possible outcomes:

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Therefore,

$$\begin{aligned}\mathbb{P}(\text{2nd flip comes up heads}) &= \mathbb{P}(\mathtt{HH}) + \mathbb{P}(\mathtt{TH}) \\ &= 0.3025 + 0.2475 = \boxed{0.5500}\end{aligned}$$