American University of Beirut
STAT 210: Elementary Statistics for Sciences
2022–2023 Fall

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Chapter 10
Estimation and hypothesis testing about two populations
Part 1

Comparing two populations

Comparing two populations

Inference about one population

So far we have been discussing inference about a parameter θ of a single population:

I. Testing hypotheses concerning θ

II. Estimating θ

The statistical evidence was in the form of a sample from the population.

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Inference about two populations

Now, we are going to discuss how to compare two populations:

I. Testing hypotheses concerning the equality of θ_1 and θ_2

II. Estimating $\theta_1 - \theta_2$

The statistical evidence will be in the form of samples from the two populations.

Comparing two populations

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- Q2 If so, by how much?

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Example (Herbal remedy for high blood pressure)

The experiment involves a 50 volunteers (the subjects) with chronic high blood pressure. The researchers randomly assign the subjects to two groups:

- The treatment group receive one glass of the herbal tea every night for a whole month.
- The control group receive placebo for the same duration.

The researchers then measure the blood pressures of the two groups and compare their averages $\bar{x}$ and $\bar{y}$.

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  Q How big a difference should be considered "significant"?

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Inference about two populations

In order to compare two populations, we often compare their corresponding parameters (such as mean, proportion, standard deviation, ...).

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H$_0$ $\theta_1 - \theta_2 = 0$	H$_0$ $\theta_1 - \theta_2 = 0$	H$_0$ $\theta_1 - \theta_2 = 0$
H$_1$ $\theta_1 - \theta_2 < 0$	H$_1$ $\theta_1 - \theta_2 \neq 0$	H$_1$ $\theta_1 - \theta_2 > 0$

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- Independent samples are two independent collections:

$$X_1, X_2, X_3, \ldots, X_{n_1} \qquad\qquad Y_1, Y_2, Y_3, \ldots, Y_{n_2}$$

  with no connections between them.

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In the example about "herbal remedy for blood pressure", the two samples are ...

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We focus on the problems of inference about the population means and about the population proportions in two difference populations.

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Inference about population means

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Q2 Are the samples large or small?

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Q3 Are the populations (approximately) normally distributed?

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- Q1 Are the samples independent or paired?
- Q2 Are the samples large or small?
- Q3 Are the populations (approximately) normally distributed?
- Q4 Do we know the population standard deviations?

Comparing two populations

Inference about population means

The procedure depends on the answers to the following questions:

- Q1 Are the samples independent or paired?
- Q2 Are the samples large or small?
- Q3 Are the populations (approximately) normally distributed?
- Q4 Do we know the population standard deviations?
- Q5 If not, are the (unknown) standard deviations the same?

Comparing two populations

Inference about population means

We consider the following scenarios:

1. independent samples, σ_1 and σ_2 known,
   - (a) normal populations
   - (b) large samples
2. independent samples, σ_1 and σ_2 unknown but $\sigma_1 = \sigma_2$,
   - (a) normal populations
   - (b) large samples
3. independent samples, σ_1 and σ_2 unknown and $\sigma_1 \neq \sigma_2$,
   - (a) normal populations
   - (b) large samples
4. paired sample,
   - (a) normal population of pair differences
   - (b) large sample

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Inference about population means

We consider the following scenarios:

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   (a) normal populations
   (b) large samples
2. independent samples, σ_1 and σ_2 unknown but $\sigma_1 = \sigma_2$,
   (a) normal populations
   (b) large samples
3. independent samples, σ_1 and σ_2 unknown and $\sigma_1 \neq \sigma_2$,
   (a) normal populations
   (b) large samples
4. paired sample,
   (a) normal population of pair differences
   (b) large sample

Inference about population proportions

We consider the following scenario:

1. large independent samples