# American University of Beirut

# MATH 307K: Introduction to Ergodic Theory

Course Syllabus (2024–2025 Spring)

**Course description:** Ergodic theory is the study of "statistical" or "long-term" properties of dynamical systems. It was originated as an attempt by mathematicians to evaluate the so-called *ergodic hypothesis* in Boltzmann's statistical mechanics (namely, the equality of time and phase averages), but has since developed into a lively branch of mathematics with broader connections with statistical physics, probability theory, information theory, combinatorics, number theory, geometry, and group theory. This course is an introduction to (a selection of) the standard aspects of ergodic theory.

#### **Tentative contents:**

- Measure-preserving transformations
- Poincaré's recurrence theorem
- Mean and pointwise ergodic theorems
- Ergodicity and mixing conditions
- Concepts from topological and symbolic dynamics
- Geometry of set of invariant measures
- Unique ergodicity and minimality
- Entropy
- Shannon-McMillan-Breiman theorem
- Measures of maximal entropy (if time permits)

## Suggested readings:

- Foundations of Ergodic Theory by M. Viana and K. Oliveira.
- Ergodic Theory (with a view towards Number Theory) by M. Einsiedler and T. Ward.
- Invitation to Ergodic Theory by C. E. Silva.
- Supplementary notes will be provided.

*Note:* The electronic versions of all three books can be accessed via the university library. You may be prompted to login with your AUB account.

### **Prerequisites:**

- Maturity in real analysis (MATH 210) and the topology of compact metrizable spaces (preferably MATH 214).
- Measure theory (MATH 303). Can be taken simultaneously. We will briefly review what we need.

Feel free to contact the instructor if you have questions.

#### Format of the course:

- Lectures (regular class times)
- Exercise sessions (to be scheduled in agreement with the participants).
- Projects (towards the end of the semester)

#### Time and location:

- MWF 9:00 am-9:50 am (January 16 April 26)
- Bliss Hall 206

**Instructor:** Siamak Taati

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#### **Assessment:**

- 40% Assignments followed by exercise sessions
- 40% Final presentation or paper
- 20% Active participation

#### **Course learning outcomes:** Upon completion of this course, you are expected to be able to

- Use notation and terminology consistently and accurately;
- Recall (or quickly rediscover) basic facts from ergodic theory;
- Reproduce (or quickly look up) the statements of central results from ergodic theory and recognize their essential hypotheses;
- Apply abstract ergodic-theoretic results and concepts to concrete settings;
- Follow mathematical arguments of ergodic-theoretic results and spot potential mistakes in them;
- Use techniques and ideas from various areas of mathematics to solve ergodic-theoretic problems and develop proof strategies at an intuitive level;
- Turn ergodic-theoretic proof strategies and sketches into rigorous mathematical proofs;
- Propose reasonable potential generalizations of ergodic-theoretic results from concrete examples to abstract setting;
- Produce counter-examples (or reasonable candidates thereof) for false ergodic-theoretic statements;
- Spot errors and inconsistencies in arguments, terminology, notations, etc. in the ergodic-theoretic context.
- Follow literature in selected aspects of ergodic theory (potentially with the help of additional literature) and grasp the main ideas;
- · Effectively communicate informal ideas and rigorous arguments from ergodic theory orally and in writing.

# Program learning outcomes addressed in the course:

- Research Methods: Analyze qualitative and/or quantitative information while conducting research related to their graduate program.
- Advanced Core Knowledge in the Discipline: Analyze core concepts, research and theories in different areas of study in their discipline.
- Scholarly Communication Skills: Demonstrate scholarly communication skills in clear, detailed and well-structured scientific written and oral formats consistent with high standards of their discipline.

**Unique Learning Needs of Students with Disabilities:** AUB strives to make learning experiences as accessible as possible. If you anticipate or experience academic barriers due to a disability (including mental health, chronic or temporary medical conditions), please inform your lecturer immediately so that we can privately discuss options. In order to help establish reasonable accommodations and facilitate a smooth accommodation process, you are encouraged to contact the Accessible Education Office: <a href="mailto:accessibility@aub.edu.lb">accessibility@aub.edu.lb</a>; +961-1-350000, x3246; West Hall, 314.

**Academic Integrity:** Please refer to AUB Student Code of Conduct, in particular to section 1.1, which concerns academic misconduct including cheating, plagiarism, in-class disruption, and dishonesty. Please be aware that misconduct is vigorously prosecuted and that AUB has a zero tolerance policy.

**Non-Discrimination:** AUB is committed to facilitating a campus free of all forms of discrimination including sex/gender-based harassment prohibited by Title IX. The University's non-discrimination policy applies to, and protects, all students, faculty, and staff. If you think you have experienced discrimination or harassment, including sexual misconduct, we encourage you to tell someone promptly. If you speak to a faculty or staff member about an issue such as harassment, sexual violence, or discrimination, the information will be kept as private as possible, however, faculty and designated staff are required to bring it to the attention of the University's Title IX Coordinator. Faculty can refer you to fully confidential resources, and you can find information and contacts at [https://www.aub.edu.lb/president/titleix]. To report an incident, contact the University's Title IX Coordinator Trudi Hodges at 01-350000 ext. 2514, or titleix@aub.edu.lb. An anonymous report may be submitted online via EthicsPoint at [http://www.aub.ethicspoint.com].