Full Name: _____

Student No: _____

Grade:

Read before you start:

- Please make sure you write your full name and student number.
- The exam consists of 7 questions, most with multiple parts, and a total score of 145 points.
- All answers require justifications. To get full credit, the justifications must be clearly written, with correct usage of mathematical notations.
- The duration of the exam is 2 hours.

You can use the remainder of this page as scratch paper.

1. (40 points) Determine which of the following statements is True and which is False. In each case, give a short justification.

_____ The function $f(x) = \dfrac{1}{x-1}$ is decreasing on its domain because $f'(x) = -\dfrac{1}{(x-1)^2}$ is negative.

_____ If a car is going backwards on a road but it is slowing down, then the graph of its position as a function of time is concave up.

_____ The function $f(x) = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}$ has no horizontal tangent line.

_____ Since the function $f(x) = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}$ is continuous on the interval $[0, 2]$, it necessarily achieves a maximum on this interval.

_____ Since the function $f(x) = x^{1/3}$ is continuous everywhere, it is also differentiable everywhere.

_____ $\dfrac{\mathrm{d}}{\mathrm{d}x}\pi^{13} = 13\pi^{12}$.

_____ $\lim_{h\to 0} \dfrac{\sin(2x+h) - \sin(2x)}{h} = \cos(2x)$.

_____ Since $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x)$, it follows that $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin(2x) = \cos(2x)$.

2. (10 points) Find the derivative of the following functions:

(a) $f(x) = \dfrac{x}{\sqrt{2-x}}$

(b) $g(x) = \sin(3x)\cos(3x)$

3. (5 points) Use the definition to calculate the derivative of $f(x) = \sqrt{x+1}$ at $x = 3$.
[*Note*: You will receive no points for using derivative rules, but you may use them to check your answer.]

4. (10 points) Find all the points on the curve described by the equation $x^2 + y^2 + xy = 3$ at which the tangent line is horizontal.

5. (10 points) Assuming $f(2) = 5$ and $f'(2) = 11$, compute the following:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{f(x)}{x^2}\right)\Big|_{x=2}$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x} f(\sqrt{x})\Big|_{x=4}$

6. (20 points) Consider the function $f(x) = x^5 + 4x + cos(\pi x)$.

(a) Show that $f(x)$ is an increasing function. [*Hint*: $\sin(\pi x) \geq -1$.]

(b) Argue that $f(x)$ has an inverse. [*Hint*: Use the MVT to argue that $y = f(x)$ uniquely determines x.]

Let $g(y)$ denote the inverse of $f(x)$.

(c) Show that $g(4) = 1$.

(d) Find $g'(4)$. [*Hint*: Set $y = f(x)$, and use implicit differentiation with respect to y to find $\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right|_{\substack{x=1\\y=4}}$.]

7. (50 points) Let $f(x) = (x+4)(x-1)(x-3) = x^3 - 13x + 12$.

   (a) Identify the domain of $f(x)$.

   (b) Identify the x-intercepts and the y-intercepts of the graph of the function.

   (c) Identify the intervals over which $f(x)$ is positive and the intervals over which it is negative.

   (d) Does the graph of $f(x)$ have any symmetries?

(e) Does the graph of $f(x)$ have any vertical or horizontal asymptotes? If so, identify them.

(f) Find the critical points of $f(x)$.

(g) Identify the intervals over which $f(x)$ is increasing and the intervals over which it $f(x)$ is decreasing.

(h) Identify the intervals over which $f(x)$ is concave up and the intervals over which it is concave down.

(i) Does the graph of $f(x)$ have any inflection point? If so, identify them.

(j) Sketch the graph of $f(x)$, indicating the information gathered above.

You can use this sheet as extra space for your solutions.

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