

Full Name: _____

Student No: _____

Grade:

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Read before you start:

- Please make sure you write your full name and student number.
- The exam consists of 10 questions, each with multiple parts, and a total score of 140 points.
- All answers require justifications. To get full credit, the justifications must be clearly written, with correct usage of mathematical notations.
- The duration of the exam is 2 hours.

You can use the remainder of this page as scratch paper.

1. (35 points) Determine which of the following statements is True and which is False. In each case, give a short justification.

_____ The two functions $f(x) = x^2$ and $g(u) = u^2$ are one and the same.

_____ The two functions $f(x) = \frac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ are one and the same.

_____ Whether or not $\lim_{x \rightarrow 1} f(x)$ exists does not depend on how $f(1)$ is defined.

_____ If $\lim_{x \rightarrow a} f(x) = 0$ and g is defined near a , then $\lim_{x \rightarrow a} f(x)g(x) = 0$.

_____ A function can cross its horizontal asymptote.

_____ $\cos(\pi/2 - x) = \sin x$ for every x .

_____ If you are currently taller than your mother, then at some time since your birth, you and your mother must have had exactly the same height.

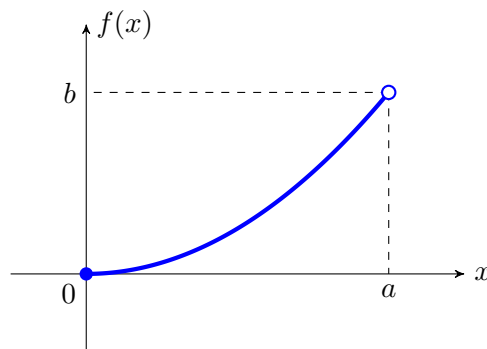
2. (10 points)

(a) Can an even function have an inverse?

[Recall: The *inverse* of a function f is a function g such that $y = f(x)$ if and only if $x = g(y)$]

(b) Suppose f is an even function and g is an odd function. Is the function $g \circ f$ even, odd, or neither?

3. (10 points) The following is the graph of a function $f(x)$.



- (a) Identify the domain and the range of f .

- (b) Sketch the graph of the function $g(x) = b - f(x + a)$.

4. (20 points) Evaluate each of the following limits or determine it does not exist.

(a) $\lim_{x \rightarrow -2} \sqrt{2x^3 + 20}$

(b) $\lim_{x \rightarrow -5} \frac{x^3 - 25x}{x^2 + 5x}$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - x}{|x - 1|}$

(d) $\lim_{x \rightarrow 0} \frac{3x^2}{(\sin 2x)^2}$

5. (10 points) Consider the function

$$f(x) = \begin{cases} \frac{\sqrt{x} - 1}{x - 1} & \text{if } x > 1, \\ 5 & \text{if } x = 1, \\ \frac{2x - 2}{x^2 + 2x - 3} & \text{if } x < 1. \end{cases}$$

Does $\lim_{x \rightarrow 1} f(x)$ exist? If yes, identify its value. If not, explain why.

6. (10 points) Evaluate each of the following limits or determine it does not exist.

(a) $\lim_{x \rightarrow +\infty} \frac{5x^2 - 2x + 1}{5x^2 + 2x + 1}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - (\sin x)^2}}{x + \sin x}$

7. (15 points) Consider the function $f(x) = \frac{x^2 + \sqrt{x+1}}{(x+1)(x+2)}$.

(a) Identify the domain of f .

(b) Identify the vertical and horizontal asymptotes of f .

8. (10 points) Consider the function $f(x) = \frac{x + \cos x}{2 + \sin x}$.

(a) Argue that f has a zero between $-\pi$ and π .

[Recall: A zero of a function f is a value a such that $f(a) = 0$.]

(b) Argue that f has a zero between -1 and 0 .

9. (10 points) Consider the function $f(x) = x\lfloor x \rfloor$.

[Recall: $\lfloor x \rfloor$ stands for the *floor* of x , that is, what we get if we round x down.]

(a) Argue that f is continuous at $x = 0$.

[Hint: Use the Sandwich Theorem.]

(b) Identify all the points at which $f(x)$ is continuous.

10. (10 points) Consider the function $f(x) = \frac{2-x}{x-1}$. We know that $\lim_{x \rightarrow \infty} f(x) = -1$.

(a) How large must x be in order to guarantee that $f(x) = -1 \pm 0.01$?

(b) Given a generic $\varepsilon > 0$, find $M > 0$ such that $|f(x) - (-1)| < \varepsilon$ whenever $x > M$.

You can use this page as extra space for your solutions.