Full Name: ______________________

Student No: ______________________

Instructor: Siamak Taati

Grade:

**Read before you start:**

- Please make sure you write your full name and student number.
- The exam consists of 9 questions, most with multiple parts, and a total score of 110 points.
- All answers require justifications. To get full credit, the justifications must be clearly written, with correct usage of mathematical notations.
- The duration of the exam is 2 hours.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

You can use the remainder of this page as scratch paper.

1. (30 points) Determine which of the following statements is True and which is False. In each case, give a short justification.

_____ If $\lim_{x\to 1} f(x) = +\infty$ and $g(x) \geq 0$ for every $x$, then $\lim_{x\to 1} f(x)g(x) = +\infty$.

_____ If the acceleration of a particle moving on a line is positive, then either its velocity is already positive, or its velocity will eventually become positive.

_____ $\int_{-1}^{1} f(x)\,\mathrm{d}x$ is an antiderivative of $f(x)$.

_____ If $F$ is a differentiable function, then we can conclude that $\displaystyle\int_0^x F'(t)\,\mathrm{d}t = F(x)$.

_____ $\displaystyle\int_{-5}^{5} x\sqrt{x^8+1}\,\mathrm{d}x = 0$ because $f(x) = x\sqrt{x^8+1}$ is an odd function.

_____ $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,\mathrm{d}x = \left.\frac{-1}{x-1}\right|_{x=0}^{2} = -2.$

2. (10 points) Evaluate the following indefinite integrals:

(a) $\displaystyle\int x(1+x^2)^{3/2}\,\mathrm{d}x.$

(b) $\displaystyle\int (\cos x)^5\,\mathrm{d}x.$ [*Hint*: Use the substitution $u = \sin x$.]

3. (10 points) Evaluate the following definite integrals:

(a) $\displaystyle\int_1^2 (1 - t^{-2})(t + t^{-1})^5 \, \mathrm{d}t.$

(b) $\displaystyle\int_{-1}^1 \frac{\sin(x)}{x^2 + 1} \, \mathrm{d}x.$ [*Hint*: Pay attention to symmetries.]

4. (10 points) Identify the vertical and horizontal asymptotes of the graph of the function

$$f(x) = \frac{(x-4)|x-\sin x|}{x^2-5x+4}\ .$$

5. (15 points) Consider the function

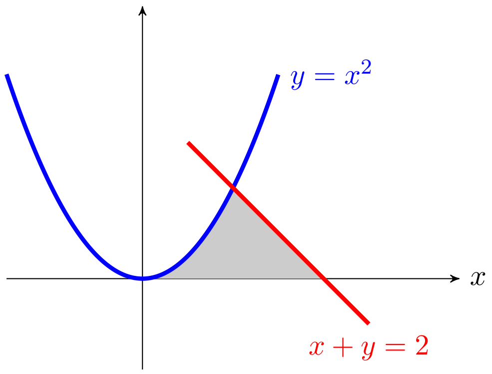
$$h(t) = \begin{cases} t^2 - 3t + 1 & \text{if } t \geq 0, \\ -t^3 + 2t + 1 & \text{if } t < 0. \end{cases}$$

(a) Verify that $h(t)$ is continuous at $t = 0$.

(b) Identify all the points in the interval $[-2, 2]$ at which $h(t)$ has a local minimum or a local maximum.

(c) Find the absolute minimum and absolute maximum of $h(t)$ over the interval $[-2, 2]$.

6. (5 points) Find the area of the shaded region in the figure.



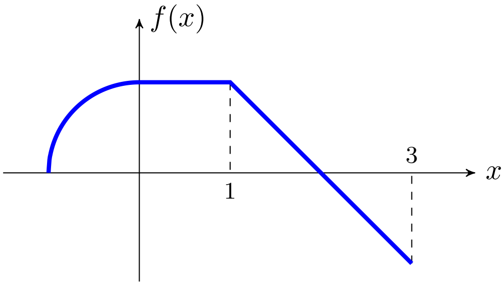

7. (5 points) Suppose $F(x) = \displaystyle\int_1^{x^2} \cos(\sqrt{t})\ \mathrm{d}t$. Compute $F'(\pi)$.

[*Hint*: Write $F(x)$ as a composition, and apply the chain rule and the Fundamental Theorem of Calculus.]

8. (10 points) Find $\int_{-1}^{3} f(x)\,\mathrm{d}x$, where

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } -1 \le x < 0, \\ 1 & \text{if } 0 \le x \le 1, \\ 2 - x & \text{if } 1 < x \le 3. \end{cases}$$

[*Hint*: Interpret the integral as a signed area.]



9. (15 points) Consider the function $f(x) = -2x^2 + 5x - 1$. Use the definition of definite integrals via Riemann sums to compute $\int_1^2 f(x)\,\mathrm{d}x$. Namely:

   (a) Partition the interval $[1, 2]$ into $n$ tiny intervals of size $\Delta x = 1/n$ and write down the corresponding Riemann sum approximating $\int_1^2 f(x)\,\mathrm{d}x$. Use the left endpoint of each tiny interval as its representative.

   (b) Compute the Riemann sum obtained in the previous part.
   [*Hint*: You can use the formulas on the front page of the exam.]

(c) Compute the integral $\int_1^2 f(x)\,\mathrm{d}x$ by taking the limit of the Riemann sum obtained in the previous parts as $n \to \infty$.

You can use this page as extra space for your solutions.