Full Name: ______________________

Student No: ______________________

Instructor: Siamak Taati

Grade:

Read before you start:

- Please make sure you write your full name and student number.
- The exam consists of 9 questions, most with multiple parts, and a total score of 110 points.
- All answers require justifications. To get full credit, the justifications must be clearly written, with correct usage of mathematical notations.
- The duration of the exam is 2 hours.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

You can use the remainder of this page as scratch paper.

1. (30 points) Determine which of the following statements is True and which is False. In each case, give a short justification.

FALSE If $\lim_{x\to 1} f(x) = +\infty$ and $g(x) \geq 0$ for every x, then $\lim_{x\to 1} f(x)g(x) = +\infty$.

Solution: (This question is similar to one of the True/False questions in the first midterm.) We cannot say much about the limit of $f(x)g(x)$ without knowing what happens to $g(x)$ as $x \to 1$. In each of the four examples below, $\lim_{x\to 1} f(x) = +\infty$ and $g(x) \geq 0$.

Example 0: Let $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = 0$ for every x. Then, $\lim_{x\to 1} f(x)g(x) = 0$.

Example 1: Let $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = (x-1)^2$. Then, $\lim_{x\to 1} f(x)g(x) = 1$.

Example 2: Let $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = |x-1|^3$. Then, $\lim_{x\to 1} f(x)g(x) = 0$.

Example 3: Let $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = |x-1|$. Then, $\lim_{x\to 1} f(x)g(x) = +\infty$.

Example 4: Let $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = (x-1)^2 \sin\left(\dfrac{1}{x-1}\right)$. Then, $\lim_{x\to 1} f(x)g(x)$ does not exist due to wild oscillations near $x = 1$.

__?__ If the acceleration of a particle moving on a line is positive, then either its velocity is already positive, or its velocity will eventually become positive.

Solution: Whether this statement is true or false depends on its precise interpretation, so this is a good example of how mathematical reasoning clarifies an otherwise ambiguous statement.

A naive reasoning would go as follows: The acceleration is the rate of change of velocity. Suppose that the current velocity of the particle is negative. If its acceleration is positive, then the velocity of the particle gradually increases until it eventually becomes positive.

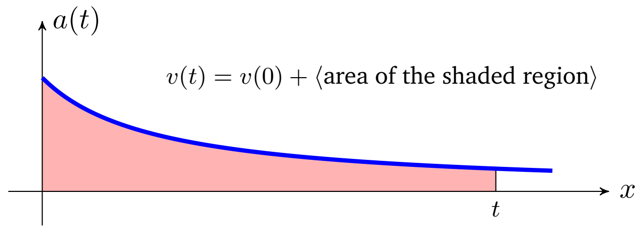
To see what is wrong with this reasoning, let us formulate it more precisely. Let $v(t)$ and $a(t)$ denote the velocity and acceleration of the particle as functions of time. Suppose that the particle initially has a negative velocity, that is, $v(0) < 0$.

By definition, $a(t) = v'(t)$, therefore by the Fundamental Theorem of Calculus, at every time $t \geq 0$, we have

$$\int_0^t a(s)\,\mathrm{d}s = v(t) - v(0)\ , \qquad \text{hence} \qquad v(t) = v(0) + \int_0^t a(s)\,\mathrm{d}s\ .$$

If the acceleration $a(t)$ is constant and positive, that is, $a(t) = a_0$ for some constant $a_0 > 0$, then $\int_0^t a(s)\,\mathrm{d}s = a_0 t$. It follows that $v(t) = v(0) + a_0 t$ grows linearly, and in particular, will be positive as soon as $t > -v(0)/a_0$.

However, in the statement, there is no mention of the acceleration being constant. What if the acceleration decreases with time while remaining positive? Could it be that the acceleration decays to zero so fast that the velocity of the particle always remains negative?



Here is an example of how this can happen: Let $v(0) = -1$ and $a(t) = \dfrac{1}{(t+1)^2}$. Then,

$$v(t) = -1 + \int_0^t \frac{1}{(s+1)^2}\,\mathrm{d}s = -1 + \left[\frac{-1}{(s+1)}\right]_{s=0}^{t} = -1 + \frac{-1}{t+1} - \frac{-1}{0+1} = -\frac{1}{t+1}\,,$$

which is always negative.

FALSE $\displaystyle\int_{-1}^{1} f(x)\,\mathrm{d}x$ is an antiderivative of $f(x)$.

Solution: The definite integral $\displaystyle\int_{-1}^{1} f(x)\,\mathrm{d}x$, if it exists, is just a fixed number, so its derivative is zero.
In contrast, $F(y) = \displaystyle\int_{-1}^{y} f(x)\,\mathrm{d}x$ is an antiderivative of $f(x)$ by the Fundamental Theorem of Calculus.

FALSE If F is a differentiable function, then we can conclude that $\displaystyle\int_0^x F'(t)\,\mathrm{d}t = F(x)$.

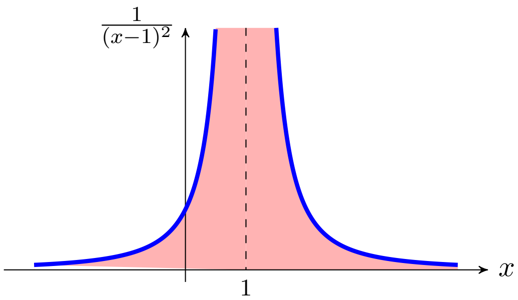
Solution: By the Fundamental Theorem of Calculus, $\displaystyle\int_0^x F'(t)\,\mathrm{d}t = F(x) - F(0)$ which differs from $F(x)$ by a possibly non-zero constant $F(0)$. Indeed, by integrating the derivative of a function $F(x)$ we can recover $F(x)$ only up to a constant.

TRUE $\displaystyle\int_{-5}^{5} x\sqrt{x^8+1}\,\mathrm{d}x = 0$ because $f(x) = x\sqrt{x^8+1}$ is an odd function.

Solution: We have $f(-x) = (-x)\sqrt{(-x)^8+1} = -x\sqrt{x^8+1} = -f(x)$, hence $f(x)$ is indeed odd. The integral of any (integrable) odd function over a symmetric interval $[-a, a]$ is zero.

FALSE $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,\mathrm{d}x = \frac{-1}{x-1}\bigg|_{x=0}^{2} = -2.$

Solution: (This question is similar to an example discussed in class.)
The function $\dfrac{1}{(x-1)^2}$ is strictly positive over its domain, so its integral (the signed area under its curve) over $[0, 2]$ cannot be negative! Still, $F(x) = \dfrac{-1}{x-1}$ is an antiderivative for $f(x) = \dfrac{1}{(x-1)^2}$, so the computation seems valid. So, where is the problem?



The problem is that $f(x)$ and $F(x)$ are not defined at $x = 1$, so the Fundamental Theorem of Calculus does not apply. (Remember, in order for the FTC to be applicable to f on $[0, 2]$, F has to be differentiable on $[0, 2]$ with $F' = f$.) On a related note, although every antiderivative of $f(x) = \dfrac{1}{(x-1)^2}$ on $[0, 1)$ differs from $F(x) = \dfrac{-1}{x-1}$ by a constant, and similarly for $(1, 2]$, the constants for $[0, 1)$ and $(1, 2]$ may be different.

Remark. The integral $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,\mathrm{d}x$ is an example of an *improper* integral, which you will learn more about in MATH 102.

2. (10 points) Evaluate the following indefinite integrals:

(a) $\displaystyle\int x(1+x^2)^{3/2}\,\mathrm{d}x.$

Solution: Let $u = 1 + x^2$ and note that $\mathrm{d}u = 2x\,\mathrm{d}x$. Hence, using the substitution method, we can write

$$\int x(1+x^2)^{3/2}\,\mathrm{d}x = \int \frac{1}{2}u^{3/2}\,\mathrm{d}u\bigg|_{u=1+x^2} = \left[\frac{1}{5}u^{5/2} + C\right]_{u=1+x^2} = \boxed{\frac{1}{5}(1+x^2)^{5/2} + C},$$

where C is a constant.

(b) $\displaystyle\int (\cos x)^5\,\mathrm{d}x.$ [*Hint*: Use the substitution $u = \sin x$.]

Solution: Following the hint, let $u = \sin x$. Note that $\mathrm{d}u = \cos x\,\mathrm{d}x$. Hence,

$$\begin{aligned}(\cos x)^5 &= (\cos x)^2 \cdot (\cos x)^2 \cdot (\cos x)\\ &= \big(1 - (\sin x)^2\big) \cdot \big(1 - (\sin x)^2\big) \cdot (\cos x)\\ &= (1-u^2)^2 \cdot \frac{\mathrm{d}u}{\mathrm{d}x}\,.\end{aligned}$$

Using the substitution method, we can thus write

$$\begin{aligned}\int (\cos x)^5\,\mathrm{d}x &= \int (1-u^2)^2\,\mathrm{d}u\bigg|_{u=\sin x}\\ &= \int (1 - 2u^2 + u^4)\,\mathrm{d}u\bigg|_{u=\sin x}\\ &= \left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C\right]_{u=\sin x}\\ &= \boxed{\sin x - \frac{2}{3}(\sin x)^3 + \frac{1}{5}(\sin x)^5 + C}\,,\end{aligned}$$

where C is a constant.

3. (10 points) Evaluate the following definite integrals:

(a) $\displaystyle\int_1^2 (1 - t^{-2})(t + t^{-1})^5 \, \mathrm{d}t.$

Solution: Let $u = t + t^{-1}$ and note that $\mathrm{d}u = (1 - t^{-2})\,\mathrm{d}t$. Therefore, by the substitution method,

$$\begin{aligned}
\int_1^2 (1 - t^{-2})(t + t^{-1})^5 \, \mathrm{d}t &= \int_{t=1}^{2} u^5 \, \mathrm{d}u \\
&= \int_{u=2}^{5/2} u^5 \, \mathrm{d}u \qquad \begin{cases} t = 1 \longrightarrow u = 1 + 1^{-1} = 2 \\ t = 2 \longrightarrow u = 2 + 2^{-1} = 5/2 \end{cases} \\
&= \frac{1}{6} u^6 \Big|_{u=2}^{5/2} \\
&= \boxed{\frac{1}{6}\left((5/2)^6\right) - 2^6} \\
&= \frac{3843}{128} = 30.0234375 \; .
\end{aligned}$$

(b) $\displaystyle\int_{-1}^1 \frac{\sin(x)}{x^2+1} \, \mathrm{d}x.$ [*Hint*: Pay attention to symmetries.]

Solution: The function $f(x) = \dfrac{\sin(x)}{x^2+1}$ is defined everywhere and is continuous, hence it is integrable on $[-1, 1]$.

Observe that $f(x)$ is an odd function because

$$f(-x) = \frac{\sin(-x)}{(-x)^2+1} = \frac{-\sin(x)}{x^2+1} = -f(x) \; .$$

Since the interval $[-1, 1]$ is also symmetric about the origin, we have $\displaystyle\int_{-1}^1 \frac{\sin(x)}{x^2+1} \, \mathrm{d}x = 0.$

4. (10 points) Identify the vertical and horizontal asymptotes of the graph of the function

$$f(x) = \frac{(x-4)|x-\sin x|}{x^2-5x+4} \ .$$

Solution: Observe that $x^2 - 5x + 4 = (x-4)(x-1)$, hence

$$f(x) = \frac{(x-4)|x-\sin x|}{(x-4)(x-1)} = \begin{cases} \dfrac{|x-\sin x|}{x-1} & \text{if } x \neq 4, \\ \text{undefined} & \text{if } x = 4. \end{cases}$$

Vertical asymptotes: Vertical asymptotes correspond to infinite limits. Since $f(x)$ is continuous over its domain, the only points at which the limit of $f(x)$ can *potentially* be $\pm\infty$ are $x = 1$ and $x = 4$.

When $x \to 1^+$, the numerator $|x - \sin x|$ approaches $|1 - \sin 1| > 0$, while the denominator $x - 1$ goes to 0 from the right. Therefore,

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{|x-\sin x|}{x-1} = +\infty \ .$$

When $x \to 1^-$, the numerator goes to the same value, but the denominator $x-1$ approaches 0 from the left. Hence,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{|x-\sin x|}{x-1} = -\infty \ .$$

We conclude that the line $x = 1$ is a vertical asymptote of $f(x)$.

On the other hands, we have

$$\lim_{x\to 4} f(x) = \lim_{x\to 4} \frac{|x-\sin x|}{x-1} = \frac{|4-\sin 4|}{4-1} \approx 1.5856 \ ,$$

so the line $x = 4$ is not a vertical asymptote of $f(x)$.

Horizontal asymptotes: Horizontal asymptotes correspond to limits at $+\infty$ and $-\infty$.

We have

$$\begin{aligned} \lim_{x\to+\infty} f(x) &= \lim_{x\to+\infty} \frac{|x-\sin x|}{x-1} \\ &= \lim_{x\to+\infty} \frac{x\left|1-\frac{\sin x}{x}\right|}{x\left(1-\frac{1}{x}\right)} && |x| = x \text{ when } x\to+\infty \\ &= \lim_{x\to+\infty} \frac{\left|1-\frac{\sin x}{x}\right|}{\left(1-\frac{1}{x}\right)} \\ &= \frac{\left|1-\lim\limits_{x\to+\infty}\frac{\sin x}{x}\right|}{\lim\limits_{x\to+\infty}\left(1-\frac{1}{x}\right)} && \text{arithmetics of limits and continuity of absolute value} \\ &= 1 \ , \end{aligned}$$

where, for the last step, we have used the fact that $\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ for positive x, hence $\lim\limits_{x\to+\infty} \frac{\sin x}{x} = 0$ by the Sandwich Theorem. It follows that the line $y = 1$ is a horizontal asymptote of $f(x)$ (as $x \to +\infty$).

When $x \to -\infty$, the only difference is that $|x| = -x$. Otherwise, we similarly obtain

$$\lim_{x\to-\infty} f(x) = -1 \ .$$

Therefore, the line $y = -1$ is a horizontal asymptote of $f(x)$ (as $x \to -\infty$).

5. (15 points) Consider the function

$$h(t) = \begin{cases} t^2 - 3t + 1 & \text{if } t \geq 0, \\ -t^3 + 2t + 1 & \text{if } t < 0. \end{cases}$$

(a) Verify that $h(t)$ is continuous at $t = 0$.

Solution: We have

$$\begin{aligned} h(0) &= 1^2 - 3 \times 0 + 1 = 1 \ , \\ \lim_{t \to 0^+} h(t) &= \lim_{t \to 0} t^2 - 3t + 1 = 1 \ , \\ \lim_{t \to 0^-} h(t) &= \lim_{t \to 0} -t^3 + 2t + 1 = 1 \ . \end{aligned}$$

Since the left and right limits of $h(t)$ as t approaches 0 exist and coincide with $h(0)$, we conclude that $h(t)$ is continuous at $t = 0$.

(b) Identify all the points in the interval $[-2, 2]$ at which $h(t)$ has a local minimum or a local maximum.

Solution: Every point at which $h(t)$ has a local minimum or a local maximum over $[-2, 2]$ belongs to one of the three categories: (i) points at which the derivative of h is zero, (ii) points at which the derivative of h does not exist, (iii) boundary points.

We have

$$h'(t) = \begin{cases} 2t - 3 & \text{if } t > 0, \\ -3t^2 + 2 & \text{if } t < 0. \end{cases}$$

At $t = 0$, the function is not differentiable because its left and right derivatives do not coincide:

$$h'_-(0) = -3 \times 0^2 + 2 = 2 \ , \qquad h'_+(0) = 2 \times 0 - 3 = -3 \ .$$

(i) Points at which $h' = 0$:

- Setting $2t - 3 = 0$, we obtain a solution $t = 3/2$, which is positive. Since $h''(3/2) = 2 > 0$, the function has a local minimum at $t = 3/2$.
- Setting $-3t^2 + 2 = 0$, we obtain two solutions $t = \pm\sqrt{2/3}$. However, we have to exclude $+\sqrt{2/3}$ because the expression $h'(t) = -3t^2 + 2$ is valid only for $t < 0$. Since $h''\big(-\sqrt{2/3}\big) = -6 \times \big(-\sqrt{2/3}\big) > 0$, the function has a local minimum at $t = -\sqrt{2/3} \approx -0.8165$.

(ii) Points at which h' does not exist:

- The only point at which h' does not exist is $t = 0$. Since $h'_-(0) = 2 > 0$ and $h'_+(0) = -3 < 0$, we conclude that the function has a local maximum at $t = 0$.

(iii) Boundary points:

- Since $h'(-2) = -3 \times (-2)^2 + 2 = -10 < 0$, over $[-2, 2]$, the function has a local maximum at $t = -2$.
- Since $h'(2) = 2 \times 2 - 3 = 1 > 0$, over $[-2, 2]$, the function has a local maximum at $t = 2$.

An alternative approach to determining whether $h(t)$ has a local maximum, a local minimum, or neither at each of the above points is to identify the monotonicity of $h(t)$ in between these points.

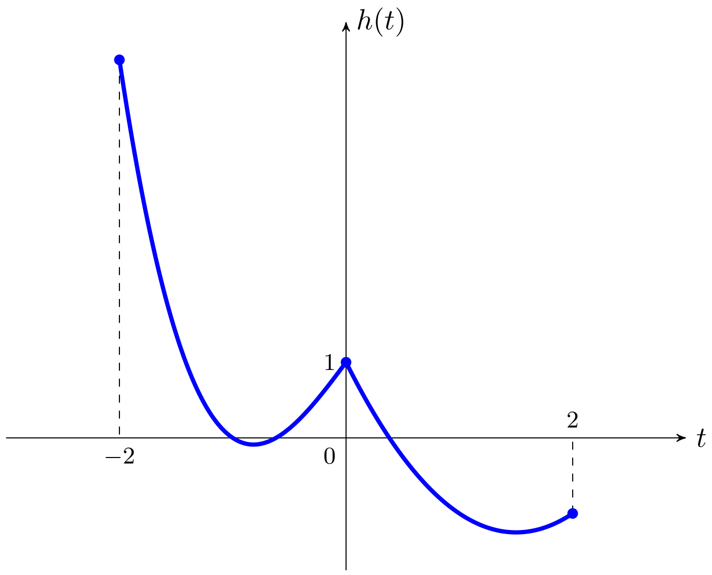
(c) Find the absolute minimum and absolute maximum of $h(t)$ over the interval $[-2, 2]$.

Solution: We compare the values of all the local minima and maxima of h over $[-2, 2]$:

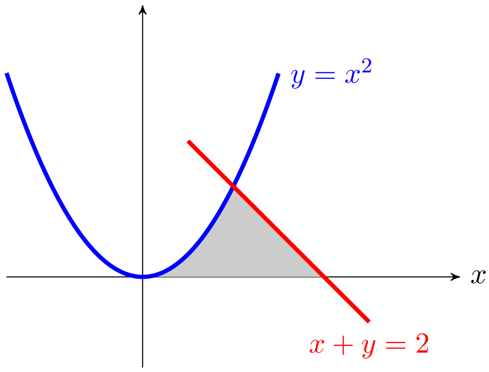
t	$h(t)$
-2	$-(-2)^3 + 2 \times (-2) + 1 = 5$
$-\sqrt{2/3}$	$-\left(-\sqrt{2/3}\right)^3 + 2 \times \left(-\sqrt{2/3}\right) + 1 \approx -0.08866$
0	1
$3/2$	$(3/2)^2 - 3 \times (3/2) + 1 = -5/4$
2	$1^2 - 3 \times 1 + 1 = -1$

Therefore,

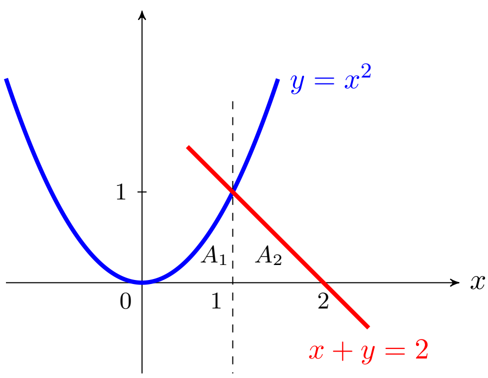
- The absolute maximum of h over $[-2, 2]$ is 5, which is attained at $t = -2$.
- The absolute minimum of h over $[-2, 2]$ is $-5/4$, which is attained at $t = 3/2$.



6. (5 points) Find the area of the shaded region in the figure.



Solution:



The line $x = 1$ (dashed line in the figure) splits the shaded region into two parts. On the left, we have a region whose area is

$$A_1 = \int_0^1 x^2 \, \mathrm{d}x = \frac{1}{3} x^3 \Big|_{x=0}^{1} = {}^1\!/_3 \; .$$

On the right, we have a isosceles right triangle with leg length 1, which has area

$$A_2 = \frac{1}{2} \times 1 \times 1 = {}^1\!/_2 \; .$$

Therefore,

$$\langle \text{area of shaded region} \rangle = A_1 + A_2 = {}^1\!/_3 + {}^1\!/_2 = \boxed{{}^5\!/_6} \; .$$

7. (5 points) Suppose $F(x) = \displaystyle\int_1^{x^2} \cos(\sqrt{t}) \, \mathrm{d}t$. Compute $F'(\pi)$.
[*Hint*: Write $F(x)$ as a composition, and apply the chain rule and the Fundamental Theorem of Calculus.]

Solution: Defining $G(y) = \displaystyle\int_1^{y} \cos(\sqrt{t}) \, \mathrm{d}t$, we can write $F(x) = G(x^2)$. By the chain rule,

$$F'(x) = 2x \, G'(x^2) \; .$$

By the Fundamental Theorem of Calculus,

$$G'(y) = \cos(\sqrt{y}) \; .$$

It follows that

$$F'(x) = 2x \cos\left(\sqrt{x^2}\right) = 2x \cos(|x|) \; .$$

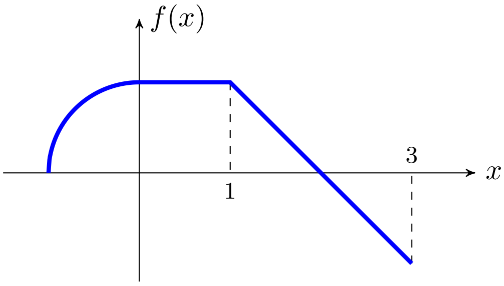
In particular,

$$F'(\pi) = 2\pi \cos(|\pi|) = -2\pi \; .$$

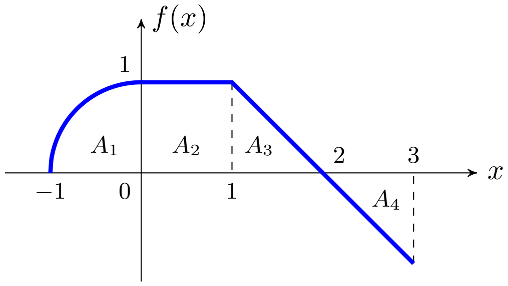
8. (10 points) Find $\int_{-1}^{3} f(x)\,\mathrm{d}x$, where

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } -1 \leq x < 0, \\ 1 & \text{if } 0 \leq x \leq 1, \\ 2-x & \text{if } 1 < x \leq 3. \end{cases}$$

[*Hint*: Interpret the integral as a signed area.]



Solution:



The value of the integral coincides with the signed area between the graph of $f(x)$ and the x-axis over the interval $[-1, 3]$. This signed area can be split into four parts as depicted in the figure. We can find each of these signed areas separately and add them up.

The first region is a quarter of the unit disk centred at the origin, hence

$$A_1 = \frac{1}{4}\pi \times 1^2 = \frac{\pi}{4}\,.$$

The second region is a unit square, thus $A_2 = 1 \times 1 = 1$. The third region is an isosceles right triangle with leg lengths 1, hence $A_3 = {}^{1}\!/_{2}$. The fourth region is likewise an isosceles right triangle with leg lengths 1, but it is below the x-axis, so its signed area is $A_4 = {}^{-1}\!/_{2}$.

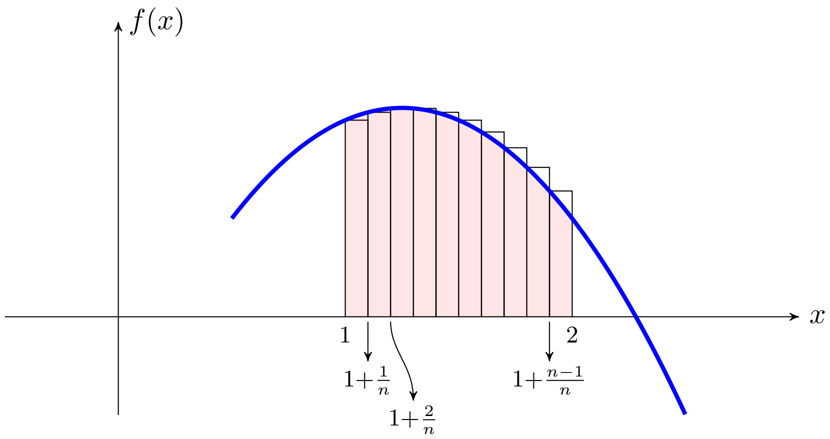
Therefore,

$$\begin{aligned} \int_{-1}^{3} f(x)\,\mathrm{d}x &= \big\langle \text{signed area between } f(x) \text{ and the } x\text{-axis over } [-1,3] \big\rangle \\ &= A_1 + A_2 + A_3 + A_4 \\ &= \frac{\pi}{4} + 1 + \frac{1}{2} - \frac{1}{2} \\ &= \boxed{1 + {}^{\pi}\!/_{4}}\,. \end{aligned}$$

9. (15 points) Consider the function $f(x) = -2x^2 + 5x - 1$. Use the definition of definite integrals via Riemann sums to compute $\int_1^2 f(x)\,\mathrm{d}x$. Namely:

(a) Partition the interval $[1, 2]$ into n tiny intervals of size $\Delta x = 1/n$ and write down the corresponding Riemann sum approximating $\int_1^2 f(x)\,\mathrm{d}x$. Use the left endpoint of each tiny interval as its representative.

Solution:



We index the tiny intervals $1, 2, 3, \ldots, n$. The kth interval is $\left[1 + \frac{k-1}{n}, 1 + \frac{k}{n}\right]$, with $c_k = 1 + \frac{k-1}{n}$ as its representative. The associated Riemann sum is

$$\begin{aligned} S_n &= \sum_{k=1}^{n} f(c_k)\Delta x \\ &= \sum_{k=1}^{n} \big(-2c_k^2 + 5c_k - 1\big)\Delta x \\ &= \sum_{k=1}^{n} \left(-2\left(1 + \frac{k-1}{n}\right)^2 + 5\left(1 + \frac{k-1}{n}\right) - 1\right) \cdot \frac{1}{n}\,. \end{aligned}$$

(b) Compute the Riemann sum obtained in the previous part.
[*Hint*: You can use the formulas on the front page of the exam.]

Solution: Expanding and simplifying the above expression, we can write

$$\begin{aligned} S_n &= \sum_{k=1}^{n} \left(\frac{-2(n+k-1)^2}{n^2} + \frac{5(n+k-1)}{n} - 1\right) \cdot \frac{1}{n} \\ &= \sum_{k=1}^{n} \frac{-2k^2 + (n+4)k + (2n^2 - n - 2)}{n^3} \\ &= \frac{-2}{n^3}\sum_{k=1}^{n} k^2 + \frac{n+4}{n^3}\sum_{k=1}^{n} k + \frac{2n^2 - n - 2}{n^3}\sum_{k=1}^{n} 1 \end{aligned}$$

Using the identities

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}\,, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}\,, \qquad \sum_{k=1}^{n} 1 = n\,,$$

we obtain

$$\begin{aligned}
S_n &= \frac{-2}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n+4}{n^3} \cdot \frac{n(n+1)}{2} + \frac{2n^2-n-2}{n^3} \cdot n \\
&= \frac{-2(n+1)(2n+1) + 3(n+4)(n+1) + 6(2n^2-n-2)}{6n^2} \\
&= \frac{11n^2+3n-2}{6n^2} .
\end{aligned}$$

(c) Compute the integral $\displaystyle\int_1^2 f(x)\,\mathrm{d}x$ by taking the limit of the Riemann sum obtained in the previous parts as $n \to \infty$.

Solution: We have

$$\begin{aligned}
\int_1^2 f(x)\,\mathrm{d}x &= \lim_{n\to\infty} S_n \\
&= \lim_{n\to\infty} \frac{11n^2+3n-2}{6n^2} \\
&= \lim_{n\to\infty} \left(\frac{11}{6} + \frac{1}{2n} - \frac{1}{3n^2} \right) \\
&= \boxed{11/6} .
\end{aligned}$$

To double-check our answer, we can use the Fundamental Theorem of Calculus to compute

$$\begin{aligned}
\int_1^2 f(x)\,\mathrm{d}x &= \int_1^2 (-2x^2+5x-1)\,\mathrm{d}x \\
&= \left[\frac{-2}{3}x^3 + \frac{5}{2}x^2 - x \right]_{x=1}^2 \\
&= \left(\frac{-16}{3} + 10 - 2 \right) - \left(\frac{-2}{3} + \frac{5}{2} - 1 \right) \\
&= 11/6 .
\end{aligned}$$